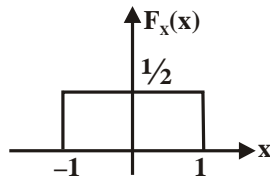


Solution of GATE-2012-Paper in Electronics&Communication-SET-D:

1. (B)



$$P\left(X \leq \frac{1}{2}\right) = \int_{-1}^{\frac{1}{2}} \frac{1}{2} dx = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

For other Random Variable:

$$P\left(Y \leq \frac{1}{2}\right) = \frac{3}{4}$$

$$\therefore P\left(\text{Max}[XY] \leq \frac{1}{2}\right) = P\left(X \leq \frac{1}{2}\right) \cdot P\left(Y \leq \frac{1}{2}\right) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

2. (A) if $x = \sqrt{-1} = i$

$$\text{Then the value of } x^x = i^i = \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)^i = e^{\left(\frac{i\pi}{2}\right)} = e^{i^2 \frac{\pi}{2}} = e^{-\pi/2}$$

$$3. (C) \quad f(z) = \frac{1}{z+1} - \frac{2}{z+3} = \frac{z+3-2z-2}{(z+1)(z+3)} = \frac{1-z}{(z+1)(z+3)}$$

Poles are $(z+1)(z+3) = 0$

$$z = -1, -3$$

Put only $z = -1$ is considered

$$\text{Res}(z = -1) = \text{Lt}_{z \rightarrow -1} (z+1) f(z)$$

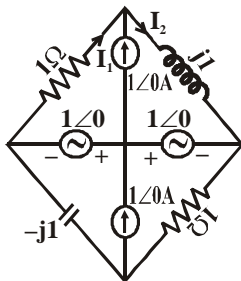
$$z \rightarrow -1$$

$$= \text{Lt}_{z \rightarrow -1} \frac{\cancel{(z+1)}(1-z)}{\cancel{(z+1)}(z+3)} = \frac{2}{2} = 1$$

$$\frac{1}{2\pi j} \int \frac{(1-z)}{(z+1)(z+3)} dz = \frac{1}{2\pi j} 2\pi j (\text{sum of periodic of poles lin within } |(z+1)|=1)$$

$$= \frac{1}{2\pi j} 2\pi j (1+0) = \frac{2\pi j}{2\pi j} = 1$$

4. (C)



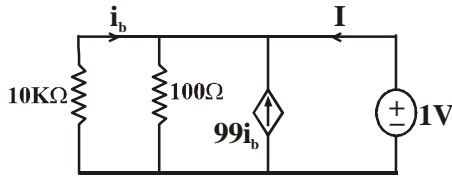
Since it is wheat stone bridge

So $I_1 \cdot 1 = I_2 \cdot j1 \rightarrow (1)$

But $I_1 + I_2 = 1 \rightarrow (2)$

$\therefore I_2 \cdot j1 + I_2 = 1 \Rightarrow I_2 = \frac{1}{1+j} A$

5. (A)



$$I = \frac{1}{10 \times 10^3} + \frac{1}{100} - 99i_b \quad \left\{ \begin{array}{l} i_b = \frac{-1}{10 \times 10^3} \end{array} \right.$$

$$= \frac{1}{10 \times 10^3} + \frac{99}{10 \times 10^3} + \frac{1}{100} = \frac{100}{100 \times 10^2} + \frac{1}{100} = \frac{2}{100} = \frac{1}{50}$$

$$R_{th} = \frac{1V}{I_0} = \frac{1}{1/50} = 50\Omega$$

6. (C) $G(s) = \frac{(s^2 + 9)(s + 2)}{(s + 1)(s + 3)(s + 4)}$

$x(t) = \sin \omega t$

$$X(s) = \frac{\omega}{s^2 + \omega^2}$$

$$Y(s) = X(s) \cdot G(s) = \frac{\omega}{s^2 + \omega^2} \cdot \frac{(s^2 + 9)(s + 2)}{(s + 1)(s + 3)(s + 4)}$$

$$e_{ss} = \lim_{s \rightarrow 0} sY(s) = \frac{s\omega}{s^2 + \omega^2} \cdot \frac{(s^2 + 9)(s + 2)}{(s + 1)(s + 3)(s + 4)}$$

= 0, if $\omega = 3$

7. (A) Prime implicants are

$\bar{x}y$ & $x\bar{y}$

x \ yz	00	01	11	10
0			(1)	(1)
1	(1)	(1)		

8. (C) $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{3}\right)^{-n} u[-n-1] - \left(\frac{1}{2}\right)^n u[n]$$

$$|z| > \frac{1}{3} \quad |z| < 3 \quad |z| > \frac{1}{2}$$

So $ROC: \frac{1}{2} < |z| < 3$

9. (A) $D = \frac{F(\theta)_{\max}}{F_{av}}$

$$F_{av} = \frac{P_{rad}}{4\pi}, \quad P_{rad} = \int_0^{2\pi} \int_0^{\pi/2} \cos^4 \theta \sin \theta d\theta d\phi$$

$$P_{rad} = 2\pi \int -t^4 dt = -2\pi \times \frac{1}{5} \cos^5 \theta \Big|_0^{\pi/2} = -2\pi \times \frac{1}{5} (0-1) = \frac{2\pi}{5}$$

$$F_{av} = \frac{1}{20\pi}, \quad F(\theta)_{max} = 1$$

$$D = 10$$

$$D(\text{in dB}) = 10 \log(10) = 10.0 \text{ dB}$$

10. No Answer is correct.

$$Z_0 = \frac{138}{\sqrt{\epsilon_r}} \log\left(\frac{b}{a}\right) = \frac{138}{\sqrt{10.89}} \log\left(\frac{2.4}{1}\right) = 41.8 \times 0.875 = 15.8 \Omega$$

11. (D) If $p_1 = p_2 = p$ then

$$H_1 = p \log_2 \frac{1}{p} + p \log_2 \frac{1}{p} + p_3 \log_2 \frac{1}{p_3} + \dots$$

If $p_1 = p + \epsilon$, $p_2 = p - \epsilon$ then

$$H_2 = (p + \epsilon) \log_2 \left(\frac{1}{p + \epsilon}\right) + (p - \epsilon) \log_2 \frac{1}{(p - \epsilon)} + p_3 \log_2 \frac{1}{p_3} + \dots$$

$$= -(p + \epsilon) \log_2 (p + \epsilon) - (p - \epsilon) \log_2 (p - \epsilon) + p_3 \log_2 \frac{1}{p_3} + \dots$$

By Comparing H_1 & H_2 value it is clear that H_2 value is decreased.

12. (A): It is a clamper circuit. Can be solved by trick also. In positive half output will be zero across diode.

And in negative it will have maximum value of -2 Volt .

So In +ve half $Y = 0$

In -ve half $Y = -2$

So only option left is

$\cos \omega t - 1$

13. (A) N-MOS:

Series \rightarrow AND

Parallel \rightarrow OR

P-MOS:

Series \rightarrow OR

Parallel \rightarrow AND

$$\text{So } Y = \overline{(A+B).C} = \overline{A} \overline{B} + \overline{C}$$

14. (D) Initial condition $x(1) = 0.5$

$$t \frac{dx}{dt} + x = t \Rightarrow \frac{dx}{dt} + \frac{1}{t} x = 1 \Rightarrow \text{It is of linear form.}$$

$$P = \frac{1}{t} \quad Q = 1$$

$$I.F = e^{\int \frac{1}{t} dt} = e^{\ln t} = t$$

Solution is given by

$$x(I.F) = \int \theta(I.F) dt + C \Rightarrow xt = \int 1.t.dt + C \Rightarrow xt = \frac{t^2}{2} + C$$

$$x(t) = 0.5 \quad \text{when } t = 1 \quad x = 0.5$$

$$1.0.5 = \frac{1^2}{2} + C \Rightarrow 1 \cdot \frac{1}{2} = \frac{1}{2} + C \Rightarrow C = 0$$

The solution is $xt = \frac{t^2}{2} \Rightarrow x = \frac{t}{2}$

15.(D) $f(t) \rightarrow F(s)$

$$t f(t) \rightarrow \frac{-dF(s)}{ds}$$

$$\frac{d}{ds} \left(\frac{1}{s^2 + s + 1} \right) = \frac{(s^2 + s + 1) \cdot 0 - 1(2s + 1)}{(s^2 + s + 1)^2}$$

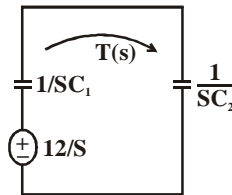
$$\frac{-dF(s)}{ds} = \frac{(2s + 1)}{(s^2 + s + 1)^2}$$

16. (B) $I_{rms} = \frac{5}{\sqrt{2}}$ & $R = 4\Omega$

$$P_{av} : I_{rms}^2 \cdot R = \left(\frac{5}{\sqrt{2}} \right)^2 \times 4 = \frac{25}{2} \times 4 = 50W$$

17. (D) $V_{C_1}(0^-) = 12V = V_{C_1}(0^+)$

$$V_{C_2}(0^-) = 0V$$



In s-domain

$$\frac{12}{S} = \left(\frac{1}{SC_1} + \frac{1}{SC_2} \right) I(s) \Rightarrow I(s) = \frac{12C_1C_2}{C_1 + C_2} \Rightarrow i(t) = \frac{12C_1C_2}{C_1 + C_2} \delta(t)$$

18. (D)
$$i = \begin{cases} \frac{V - 0.7}{500} A ; V \geq 0.7V \\ 0A ; V < 0.7V \end{cases}$$

$$i = \frac{V - 0.7}{500} A \Rightarrow V = 500i + 0.7$$

$$i = \frac{10 - V}{1000} = \frac{10 - 500i - 0.7}{1000}$$

$$1000i = 10 - 500i - 0.7 \Rightarrow 1500i = 9.3 \Rightarrow i = \frac{9.3}{1500} = 6.2 \text{ mA.}$$

25. (C) $r_{\pi} = \frac{h_{fe}}{g_m} = \frac{h_{fe}}{I_c} \times V_T \Rightarrow r_{\pi} = \frac{V_T}{i_b} = \frac{26mV}{1.1mA} \approx 25\Omega$

26. (B) $Z = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 25-6 & 15 \\ -10 & -6 \end{bmatrix} = \begin{bmatrix} 19 & 15 \\ -10 & -6 \end{bmatrix}$$

$$A^3 = A^2 - A = \begin{bmatrix} 19 & 15 \\ -10 & -6 \end{bmatrix} \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -95+30 & -57 \\ 50-12 & 30 \end{bmatrix} = \begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix}$$

$$A^3 = 19 \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} + 30 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -95 & -57 \\ 38 & 0 \end{bmatrix} + \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} -65 & -57 \\ 38 & 30 \end{bmatrix}$$

27. (C) $f(x) = x^3 - 9x^2 + 24x + 5$

$$f'(x) = 3x^2 - 18x + 24$$

$$f'(x) = 0 \Rightarrow 3x^2 - 18x + 24 = 0 \Rightarrow x^2 - 6x + 8 = 0 \Rightarrow x^2 - 2x - 4x + 8 = 0$$

$$x(x-2) - 4(x-2) = 0$$

$$(x-2)(x-4) = 0 \Rightarrow x = 2, 4$$

But 2, 4 lies inside the interval

$$f''(x) = 6x - 18$$

at $x = 2$

$$f''(x) = 12 - 18 = -6 < 0 \text{ max at } x = 2$$

But at $x = 4$

$$f''(x) = 24 - 18 = 6 > 0 \text{ min at } x = 4$$

But $f(1) = 1^3 - 9 \cdot 1^2 + 24 \cdot 1 + 5 = 1 - 9 + 24 + 5 = 30 - 9 = 21$

$$f(2) = 2^3 - 9 \cdot 2^2 + 24 \cdot 2 + 5 = 8 - 36 + 48 + 5 = 8 + 12 + 5 = 25$$

$$f(6) = 6^3 - 9 \cdot 6^2 + 24 \cdot 6 + 5 = 216 - 324 + 144 + 5 = 365 - 324 = 41$$

So the maximum value is 41.

28. (A) Current in 2 ohm resistor between A&B will be 3 Amp only. Current entering at node D will be also 3 A only. So current in 1 ohm resistor will be 5 Amp only. So $V_D - V_c = 5\text{Volt}$

29. (D) $A_v = \frac{-R_f}{R_s} = \frac{-100}{10} = -10$

$$|A_v| = 10$$

30. (D) $y = A \odot Q$

31. (A) $y(n) = g(n) \otimes h(n); h(n) = \left(\frac{1}{2}\right)^n u(n);$

By putting values

$$y(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$\Rightarrow g(n) = \delta(n) \Rightarrow \boxed{g(1) = 0}$$

32. (B) It is a high pass filter with R_1 and C .

33. (D)

$$\frac{m\pi}{a} = \frac{m\pi}{3} = 2.094 \Rightarrow m = 2$$

$$\frac{n\pi}{b} = \frac{n\pi}{1.2} = 2.618 \Rightarrow n = 1$$

So Mode will be TE_{21}

Just Calculate cut-off frequency for TE_{21} mode and

$$\omega = 6.28 \times 10^{10} \text{ is given}$$

Here $f < f_c$

So No wave propagation and hence $V_p = 0$

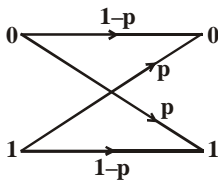
34. (B) $\theta_i = \omega_c t + k_f \int_0^t m(t) dt \Rightarrow \Delta\theta = k_f \int_0^t m(t) dt \rightarrow FM$

$$\theta_i = \omega_c t + k_p m(t) \Rightarrow \Delta\theta = k_p m(t) \rightarrow PM$$

so $k_p m(t) = 2\pi k_f \int_0^t m(t) dt$ (Due to radian unit)

$$\Rightarrow \frac{k_p}{k_f} = \frac{2\pi \left(\int_0^t m(t) dt \right)_{\max}}{m(t)_{\max}} = \frac{2\pi \times 4}{2} = 4\pi$$

35. (B)



$$P_{10} = \frac{P(y=1)}{P(x=0)} = p$$

$$P_{01} = \frac{P(y=0)}{P(x=1)} = p$$

$$P(x=0) = 9/10$$

$$P(y=1) = 1/8 \times 9/10$$

$$P(x=1) = 1/10$$

$$P(y=0) = 1/10 \times 1/8$$

$$P[Y/X] = \begin{Bmatrix} 1-p & p \\ p & 1-p \end{Bmatrix}$$

Here error means o/P Y will have 1 & i/p sent at X is 0

$$\text{So } P_e = P(X=0) \cdot (1-p)$$

$$= \frac{9}{10} \times \frac{7}{8} = \frac{63}{80}$$

36. (A) Here M_1 will be in linear region then M_2 will be either in cut-off or saturation

V_{in}	V_{out}	M_1	M_2
$< 1V$	V_{OH}	Cut off	Linear
V_{LL}	High (Variable)	Satur	Linear

$$y(s)(s^2 + 2s + 1) = -(2s + 3)$$

$$y(s) = \frac{-(2s + 3)}{(s + 1)^2} = \frac{A}{s + 1} + \frac{B}{(s + 1)^2}$$

$$B = -(2s + 3)\Big|_{s=-1} = -(-2 + 3) = -1$$

$$A = -\frac{d}{ds}(2s + 3)\Big|_{s=-1} = -2$$

$$y(t) = -2e^{-t} - 1te^{-t}$$

$$\frac{dy(t)}{dt} = 2e^{-t} - [t \cdot e^{-t} \cdot 1 + e^{-t}] = 2e^{-t} + te^{-t} - e^{-t}$$

$$\frac{dy(t)}{dt}\Big|_{t=0^+} = 2 + 0 - 1 = 1$$

40. (A)

41. (D) For controllability

$$\text{Matrix } y(c) = [B : AB : A^2B]$$

Because order of A is 3×3

$$y(c) = \begin{bmatrix} 0 & : & 0 & : & 0 & 0 & a_1a_2 \\ 0 & : & a_2 & : & a_2a_3 & 0 & 0 \\ 1 & : & 0 & : & 0 & a_1a_3 & 0 \end{bmatrix}_{3 \times 5}$$

For controllable rank of y(c) matrix should be (3)

(a) $a_1 \neq 0; a_2 = 0; a_3 \neq 0 \Rightarrow$ then rank of y(c) = 2

(b) $a_1 = 0; a_2 \neq 0; a_3 \neq 0 \Rightarrow$ rank of y(c) = 2

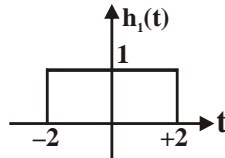
(c) $a_1 = 0; a_2 \neq 0; a_3 = 0 \Rightarrow$ Rank of y(c) = 2

(d) $a_1 \neq 0; a_2 \neq 0; a_3 = 0 \Rightarrow$ Rank y(c) = 3

42. (C) $H(j\omega) = \frac{2 \cos \omega \cdot \sin 2\omega}{\omega}$

$$H_1(j\omega) = \int_{-2}^2 2 \cdot e^{-j\omega t} dt$$

$$H_1(j\omega) = \frac{e^{-2j\omega} - e^{+2j\omega}}{-j\omega} = \frac{2 \sin 2\omega}{\omega}$$



$$h(t) = \frac{h_1\left(t - \frac{1}{2}\right) + h_1\left(t + \frac{1}{2}\right)}{2}$$

$$h(0) = \frac{h_1\left(-\frac{1}{2}\right) + h_1\left(\frac{1}{2}\right)}{2}$$

$$= \frac{1 + 1}{2} = 1$$

43. (A) $1 + G(s)H(s) = 0$

$$s^3 + as^2 + (2 + K)s + k + 1 = 0$$

$$\left. \begin{array}{l} s^3 \quad 1 \quad 2+K \\ s^2 \quad a \quad 1+K \\ s^1 \quad a(2+K) - (1+K)0 \\ s^0 \quad 1+K \end{array} \right\} \begin{array}{l} \text{for oscillatory system} \\ s^1 \text{ Row} = 0 \\ a = \frac{1+K}{2+K} \end{array} \quad \text{---(1)}$$

$$as^2 + 1 + K = 0$$

$$-a\omega^2 + (1 + K) = 0$$

$$-4a + (1 + K) = 0; \quad a = \frac{1+K}{4} \quad \text{---(2)}$$

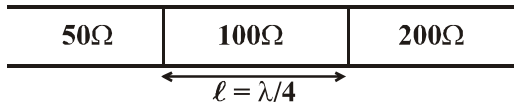
From (1) & (2)

$$\frac{1+K}{4} = \frac{1+K}{2+K} \Rightarrow 2+K = 4; \quad \boxed{K = 2}$$

$$a = \frac{2+1}{4} = \frac{3}{4} = 0.75$$

44. (D) It is time variant as well as unstable.

45. (C)



$$Z_i = \frac{Z_0^2}{Z_R} \quad \therefore Z_0 = \sqrt{Z_i \cdot Z_R}$$

So length of T/m line will be order of $\lambda/4$

$$\text{For } 429 \text{ MHz} \rightarrow \lambda = \frac{3 \times 10^8}{4.29 \times 10^8} = 0.6993 \text{ met}$$

$$\text{For } 1 \text{ GHz} \rightarrow \lambda = \frac{3 \times 10^8}{1 \times 10^9} = 0.3 \text{ met}$$

Possible lengths for 429 MHz will be:

$$0.6993/4 = 0.175 \text{ (Basic length)}$$

$$0.175, 0.525, 0.875, 1.225, 1.575, \dots$$

Possible lengths for 1MHz will be:

$$0.3/4 = 0.075 \text{ (Basic length)}$$

$$0.075, 0.225, 0.375, 0.525, 0.675, 0.825, 0.975, 1.125, 1.275, 1.425, 1.575, \dots$$

So length of desired transmission line will be 1.575 m

46. (B)

P_e (min^m) for BPSK

$$= \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_o} \cos^2 \phi} = Q \sqrt{\frac{2E_b}{N_o} \cos^2 \phi}$$

Here $\phi = 45^\circ$

$$P_e (\text{min}^m) = Q \sqrt{\frac{2E_b}{N_o} \times \frac{1}{2}} = Q \sqrt{\frac{E_b}{N_o}}$$

47. (D) $n_i = 10^{10}/\text{cm}^3$

Dopant density = $10^{19}/\text{cm}^3$

So value of n will be greater than 10^{19}

$$n \cdot p = n_i^2 = 10^{20}$$

$$p = \frac{10^{20}}{10^{19}} = 10$$

Within this volume vale of holes will be neglected.

48. (A)

Source body capacitance

$$C_{sb} = \frac{\epsilon_{si} \cdot l_s \cdot w}{l_{depletion}} = \frac{11.7 \times 8.9 \times 10^{-12} \times 0.2 \times 10^{-6} \times 1 \times 10^{-6}}{10 \times 10^{-9}} = 2 \times 10^{-15} F = 2 fF$$

$d \rightarrow$ width of gate drain source overlap

$l_s \rightarrow$ Length of source

49. (A)

Gate source overlap capacitance

$$C_{ov} = \frac{\epsilon_{ox} \cdot d \cdot w}{t_{ox}}$$

$d \rightarrow$ width of gate drain source overlap

$w \rightarrow$ width of transistor

$t_{ox} \rightarrow$ oxide thickness

$$C_{ov} = \frac{3.9 \times 8.9 \times 10^{-12} \times 20 \times 10^{-9} \times 10^{-6}}{1 \times 10^{-9}} = 694.2 \times 10^{-18} F$$

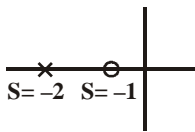
$$= 0.694 \times 10^{-15} F$$

$$= 0.7 fF (1 fF = 10^{-15} F)$$

50. (B)

51. (C)

52. (A)



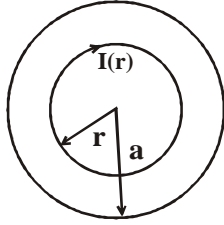
Zero is nearer to origin in case of phase lead compensator

$$Gc(s) = \frac{s+1}{s+2}$$

53. (A) $\omega_m = \frac{1}{T\sqrt{\alpha}}$; $\alpha = \frac{1}{2}$; $T = 1$ in given Queⁿ

$$\omega_m = \frac{1}{T\sqrt{1/2}} = \sqrt{2} \text{ rad/sec.}$$

54. (C) If $r < a$



$$\oint \vec{H} \cdot d\vec{l} = I(r)$$

$$\text{Here } I(r) = \frac{I \times \pi r^2}{\pi a^2}$$

$$H \cdot 2\pi r = \frac{I \times \pi r^2}{\pi a^2}$$

$$H = \frac{I \times r}{2\pi a^2}, \quad r < a$$

if $r > a$

$$\oint \vec{H} \cdot d\vec{l} = I$$

$$H \times 2\pi r = I$$

$$H = \frac{I}{2\pi r}, \quad r > a$$

55. C

56. (B)

57. (B)

58. (D) $(1.001)^{1259} = 3.52$

$$(1.001)^{2062} = 7.85$$

$$(1.001)^{1259} \cdot (1.001)^{2062} = (1.001)^{3321} = 3.5 \times 7.85 = 27.632$$

59. (C)

60. (D)

61. (D) % of the monthly budget not spent on saving = $\frac{10500 - 1500}{10500} \times 100 = 86\%$

62. (C)

63. (A)

64. (A) Let number of

Let 20 note = x

10 note = y

$$x + y = 14$$

$$20x + 10y = 230 \quad \text{---(1)}$$

$$20x + 20y = 280 \quad \text{---(2)}$$

$$-10y = -50: \quad \boxed{y = 5}$$

65. (A)